Modular Partitioning of Combinatorial State Space in Neural Systems

A Pascalian Framework for Constraint, Entropy, and Coherence

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Abstract

The human brain operates within a vast combinatorial space of possible neural states, yet exhibits stable and coherent dynamics. This paper proposes a mathematical framework in which Pascal's triangle models combinatorial expansion, while modular transformations partition state space through constraint-based filtering. The total number of possible configurations scales exponentially (2n), but modular structure selectively retains subsets of configurations, effectively organizing the space into regions of differing accessibility. Entropy derived from these partitions characterizes regimes of stability, flexibility, and transition.

This paper argues that neural systems operate near a combinatorial boundary where possible configurations approach the scale of realizable interactions, requiring constraint-driven organization. Within this regime, coherence emerges through structured sampling rather than exhaustive exploration. This framework provides a unified account of neural dynamics consistent with attractor states and low-dimensional manifolds, supporting a constraint-based interpretation of emergence in which organized behavior arises from the regulation of combinatorial complexity.

I. Problem

Combinatorial Explosion in Neural Systems

The human brain operates within an extraordinarily large space of possible activity patterns. At any given moment, each neuron can be approximated as participating in a set of discrete or graded states, and across large populations of neurons this gives rise to a combinatorial explosion in the number of potential configurations. Even under simplified binary assumptions, the number of possible system states grows exponentially with the number of elements, resulting in a state space that rapidly exceeds the system's capacity for exhaustive exploration (Mitchell, 2009).

Despite this immense combinatorial capacity, neural systems exhibit stable, low-dimensional patterns of activity associated with perception, cognition, and behavior. Empirical studies have shown that neural population activity is often constrained to structured manifolds embedded within a much higher-dimensional space of possibilities (Cunningham & Yu, 2014). This discrepancy between the size of the possible state space and the subset of states that are actually realized raises a fundamental problem: how does the brain regulate combinatorial complexity to produce coherent and functionally meaningful dynamics?

One implication is that neural systems must operate under strong constraints that limit the accessible region of state space. Without such constraints, activity would be effectively random and incoherent. Thus, the central challenge is not the generation of possibilities, but their **selective organization**, such that stable and adaptive patterns can emerge within an otherwise intractable combinatorial landscape.

A useful mathematical lens for understanding this tension is provided by the relationship between combinatorics and scaling laws. In Pascal's triangle, the total number of possible configurations for a system of n binary elements is given by the sum of row n, equal to 2^n. This provides a canonical model of combinatorial growth. When compared against biological scales, an interesting numerical alignment emerges: rows n=50 to 52 correspond to totals of approximately $2^{50} \approx 1.13 \times 10^{15}$, $2^{51} \approx 2.25 \times 10^{15}$, and $2^{52} \approx 4.50 \times 10^{15}$. These values fall within the same order of magnitude as commonly cited estimates of total synapse counts in the human brain ($\sim 10^{14}$–10^{15}).

Importantly, these quantities represent fundamentally different objects: 2^n counts the number of possible configurations of a system with n binary degrees of freedom, whereas synapse counts represent the number of physical connections in the brain. However, the numerical correspondence suggests a potentially meaningful

interpretation. Rather than behaving as a system with 10^{14}–10^{15} independent elements, the brain may operate with a much smaller number of effective degrees of freedom, constrained by structure, correlation, and dynamics. In this view, the brain's activity may be organized within a reduced-dimensional space whose effective capacity is closer to that of a system with on the order of 50–52 independent binary variables.

This interpretation is consistent with empirical findings that neural activity occupies low-dimensional manifolds despite arising from extremely high-dimensional biological substrates. It also reframes the combinatorial explosion problem: the brain does not attempt to explore its full theoretical state space, but instead compresses and organizes it through constraints that drastically reduce its effective dimensionality.

Estimates of the total number of neurons in the human brain further illustrate the uncertainty and scaling structure underlying these considerations. While a commonly cited figure is approximately **86 billion neurons**, recent analysis suggests that this value is not well constrained, with plausible estimates ranging from approximately **61 to 99 billion neurons** (Goriely, 2025). When these estimates are mapped onto cumulative Fibonacci scaling (see Appendix I), they align within a narrow discrete scaling band corresponding to Pascal rows 50–52, with empirical estimates mapping specifically to rows 50–51. Specifically, neuron estimates of ~61B, ~86B, and ~99B map to cumulative Fibonacci values corresponding to Pascal rows 50–51 (0-indexed), whose total combinatorial capacities are on the order of 10^{15}, while row 52 represents the next higher discrete level.

This relationship is summarized in **Table 1 (see Appendix I)**, which maps neuron estimates to their nearest cumulative Fibonacci values and corresponding Pascal rows:

Table 1. Mapping of Neuron Estimates to Fibonacci Scaling and Pascal Row Combinatorial Capacity (see Appendix I)

Brain Estimate (Neurons)	Corresponding Pascal Row (0-indexed)	Corresponding Pascal Row (1-indexed)	Total Combinatorial Capacity (2^n)	Approximate Magnitude
~61 billion	50	51	2^{50}	1.13×10^{15}
~86 billion	51	52	2^{51}	2.25×10^{15}
~99 billion	51	52	2^{51}	2.25×10^{15}

Notably, while rows 50–52 define the relevant scaling band, current neuron estimates do not map to row 52 directly, indicating that the brain's observed scale lies just below the next combinatorial threshold.

While this alignment does not imply a direct mechanistic equivalence between neuron counts and combinatorial state spaces, it reinforces the idea that large-scale biological complexity may organize around discrete scaling regimes rather than continuous, unstructured growth.

Taken together, these observations suggest that the brain resolves combinatorial explosion not by brute-force exploration, but through structured constraint, dimensionality reduction, and hierarchical organization. Understanding the nature of these constraints—and how they give rise to effective low-dimensional dynamics—remains a central problem in neuroscience and complex systems theory.

II. Pascal Framework

Combinatorial Expansion and Binomial Structure

To formalize the relationship between combinatorial expansion and constraint, this paper adopts Pascal's triangle as a minimal mathematical framework. Pascal's triangle provides a complete representation of binomial expansion, capturing both the total number of possible configurations and their internal structure (Graham, Knuth, & Patashnik, 1994).

At a given depth *n*, the total number of possible configurations is given by:

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

This expression defines the full combinatorial capacity of a system composed of nnn elements under binary branching conditions. The exponential growth of 2n illustrates the rapid expansion of possible states as system size increases, providing a formal representation of the combinatorial explosion described in neural systems.

However, Pascal's triangle does more than enumerate possibilities; it also defines how these possibilities are distributed. The binomial coefficients determine the number of configurations associated with each level of selection, producing a structured distribution rather than a uniform one. Values are minimal at the edges and maximal near the center, reflecting a balance between inclusion and exclusion across the system.

Within the present framework, this structure is interpreted as a map of **constraint distribution** across the combinatorial space. While 2n defines the total number of possible configurations, the binomial coefficients specify how these configurations are weighted and organized. This distinction is critical: complex systems are not defined solely by the size of their state space, but by the structured constraints that determine which regions of that space are accessible.

By providing both a measure of combinatorial capacity and an internal organization of that capacity, Pascal's triangle offers a minimal and tractable model for understanding how neural systems manage the tension between exponential possibility and constrained realization.

III. Modular Transformations

Partitioning of Combinatorial State Space

While Pascal's triangle defines the full combinatorial space of possible configurations, additional structure emerges when its coefficients are examined under modular transformations. In modular arithmetic, each coefficient $\binom{n}{k}$ is reduced modulo an integer m, producing a transformed representation of the triangle:

$$\binom{n}{k} \backslash bmodm$$

These transformations reveal systematic patterns that are not apparent in the original triangle, including periodicity, symmetry, and fractal structure (Wolfram, 2002). For example, under modulo 2, Pascal's triangle reduces to a binary pattern that forms the Sierpiński triangle, a well-known fractal exhibiting recursive self-similarity across scales.

Within the present framework, modular transformations are interpreted as defining **filters on combinatorial state space**. Rather than altering the total number of possible configurations (2n), modular reduction partitions this space into subsets of configurations that are either retained or suppressed under a given modulus. Each modulus mmm therefore induces a distinct partition of the state space, characterized by specific patterns of persistence and elimination.

From this perspective, modular structure provides a minimal mathematical model of **constraint-based selection**. For instance, modulo 2 implements a binary filter that distinguishes between active and suppressed configurations, while higher moduli introduce increasingly complex partitions that differentiate among multiple classes of states. These transformations do not reduce the underlying combinatorial capacity but instead reorganize it, producing structured subsets that can be interpreted as accessible regions within the broader state space.

Such partitioning is directly relevant to neural systems, where only a fraction of theoretically possible activity patterns is realized. Neural dynamics are shaped by anatomical connectivity, synaptic weights, and regulatory processes that constrain which configurations are accessible at any given time. In this context, modular

transformations provide an abstract representation of how constraint can structure combinatorial possibilities into functionally meaningful subsets.

IV. Entropy Structure

Stability, Flexibility, and Transition in Neural Dynamics

The partitioning of combinatorial space induced by modular transformations gives rise to measurable differences in the distribution of states. These differences can be quantified using entropy, which provides a measure of the variability or unpredictability of a system (Shannon, 1948). Within this framework, entropy characterizes how evenly configurations are distributed across the partitions defined by a given modulus.

Three broad regimes can be identified:

- **Low entropy**: Configurations are highly regular and predictable, with strong constraints limiting variability.
- **Intermediate entropy**: Configurations exhibit a balance between order and variability, allowing both stability and flexibility.
- **High entropy**: Configurations are highly variable and less predictable, reflecting reduced constraint and increased exploration of the state space.

These regimes can be interpreted in relation to neural dynamics. Low-entropy states correspond to **stable attractor configurations**, in which neural activity is highly structured and reproducible. Such states are associated with persistent representations, including memory and sustained perception (Deco, Jirsa, & McIntosh, 2011). High-entropy states, by contrast, reflect **transitional or exploratory dynamics**, in which neural activity becomes more variable and less constrained, enabling adaptation and reorganization.

Intermediate entropy represents a critical regime in which systems maintain both stability and flexibility. Empirical studies suggest that neural systems operate within such regimes, balancing integration and variability to support adaptive behavior (Cunningham & Yu, 2014; Mitchell, 2009). This balance allows the system to respond to changing inputs while preserving coherent structure.

Within the present framework, entropy is not treated as a purely statistical measure, but as an indicator of how combinatorial possibilities are distributed under constraint. Modular transformations define the partitions of state space, and entropy quantifies the degree to which these partitions are structured or diffuse. Together, they provide a formal account of how neural systems navigate the tension between combinatorial expansion and constrained realization.

V. Neural Interpretation

Attractors, Transitions, and Stability

The modular and entropic structure described in the preceding sections provides a basis for interpreting neural dynamics in terms of attractors, transitions, and stability. Neural systems operate within a high-dimensional state space defined by the activity of large populations of neurons, yet empirical evidence indicates that this activity is constrained to structured, low-dimensional manifolds (Cunningham & Yu, 2014). Within this constrained space, neural dynamics can be understood as trajectories that evolve toward and between attractor states.

Attractors correspond to stable patterns of neural activity that persist over time and resist perturbation. These states are associated with coherent functional processes, including perception, memory, and decision-making (Deco, Jirsa, & McIntosh, 2011). In the present framework, attractors correspond to regions of low entropy within the modularly partitioned state space, where configurations are highly constrained and reproducible.

Transitions occur when the system moves between attractor states, often in response to external input or internal fluctuations. These transitions are characterized by increased variability and correspond to higher-entropy regions of the state space. Such dynamics enable flexibility and adaptation, allowing the system to reconfigure its activity in response to changing conditions.

Stability arises from the balance between constraint and variability. Systems that are overly constrained may become rigid and unable to adapt, while systems with insufficient constraint may exhibit disorganized or incoherent activity. Neural systems maintain stability by operating within an intermediate regime in which attractors are sufficiently robust to support coherent function, yet transitions remain possible (Mitchell, 2009; Strogatz, 2015).

Within this framework, neural dynamics are not understood as arbitrary trajectories through a combinatorial space, but as structured movements within a partitioned and constrained landscape defined by modular filtering and entropy distribution.

VI. Integration with Pressure Model

Filtering and Structural Density

The modular partitioning of combinatorial space can be integrated with the pressure field formulation introduced earlier to provide a unified account of neural

dynamics. In this combined framework, modular transformations define the **filtering layer** that determines which configurations are retained, while the pressure field quantifies the **density of realizable structure** within those filtered regions.

Formally, the pressure at a given coordinate (n, k) is defined as:

$$P(n,k) = \binom{n}{k} \cdot \left[\binom{n}{k} \backslash bmodm\right]$$

where the modular term acts as a selection function and the binomial coefficient determines the magnitude of combinatorial convergence. This formulation ensures that only configurations that persist under modular constraint contribute to the pressure field.

Within neural systems, this relationship can be interpreted as follows:

- **Modular structure (filtering layer)**: defines which activity patterns are accessible based on underlying constraints
- **Pressure (density)**: reflects the concentration of realizable configurations within those accessible regions

Regions of high pressure correspond to areas of the state space where many pathways converge and are preserved under constraint. These regions are associated with stable attractor states and coherent neural activity. Conversely, regions of low pressure reflect either sparse convergence or suppression through filtering, corresponding to unstable or transient configurations.

This integration provides a mechanistic account of how neural systems manage combinatorial complexity. Modular filtering restricts the accessible state space, while pressure organizes the remaining configurations into regions of varying structural density. Together, these processes produce a landscape in which stable, flexible, and transitional dynamics can coexist.

VII. Implications

Weak Emergence, Coherence, and Meaning

The combined framework of combinatorial expansion, modular filtering, entropy structure, and pressure dynamics has several implications for theories of emergence, coherence, and meaning in neural systems.

First, the results support an account of **weak emergence**, in which higher-level structure arises from the organization of lower-level interactions without requiring fundamentally new ontological principles (Mitchell, 2009). Within this framework, coherent neural dynamics are understood as the outcome of constraint-based

selection within a combinatorial state space, rather than as the manifestation of irreducible higher-level properties.

Second, **coherence** is reframed as a property that emerges at the boundary between combinatorial capacity and constraint. Neural systems do not achieve coherence by maximizing the exploration of possible states, but by regulating this exploration through structured constraints. Coherent activity corresponds to regions of the state space where pressure is high and entropy is appropriately balanced, allowing stable yet flexible patterns to persist.

Finally, **meaning** can be interpreted as the experiential correlate of sustained structural coherence under constraint. Rather than being inherent to individual configurations, meaning arises from the persistence of structured relationships across time. In neural terms, this corresponds to the maintenance of stable patterns of activity that integrate information across multiple scales and temporal intervals.

This interpretation provides a non-teleological account of meaning grounded in the dynamics of complex systems. Meaning is not imposed externally or derived from intrinsic purpose; it emerges from the system's ability to maintain coherence within a constrained combinatorial space. In this sense, meaning reflects the successful regulation of complexity, enabling structured patterns to persist despite the underlying expansion of possibilities.

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Appendix

Table A2: Cumulative Fibonacci Values

Pascal Row (0-indexed)	Pascal Row (1-indexed)	Cumulative Fibonacci ΣFn	Years (Cumulative)
0	1	1	0.00
1	2	2	0.01
2	3	4	0.01
3	4	7	0.02
4	5	12	0.03
5	6	20	0.05
6	7	33	0.09
7	8	54	0.15
8	9	88	0.24
9	10	143	0.39
10	11	232	0.64
11	12	376	1.03
12	13	609	1.67
13	14	986	2.70
14	15	1,596	4.37
15	16	2,583	7.08
16	17	4,180	11.45
17	18	6,764	18.53
18	19	10,945	29.99
19	20	17,710	48.52
20	21	28,656	78.51
21	22	46,367	127.03
22	23	75,024	205.5
23	24	121,392	332.6
24	25	196,417	538.1
25	26	317,810	870.7
26	27	514,228	1,409
27	28	832,039	2,280
28	29	1,346,268	3,688
29	30	2,178,308	5,968
30	31	3,524,577	9,656

Pascal Row (0-indexed)	Pascal Row (1-indexed)	Cumulative Fibonacci ΣFn	Years (Cumulative)
31	32	5,702,886	15,624
32	33	9,227,464	25,281
33	34	14,930,351	40,905
34	35	24,157,816	66,186
35	36	39,088,168	107,091
36	37	63,245,985	173,277
37	38	102,334,154	280,368
38	39	165,580,140	453,644
39	40	267,914,295	734,012
40	41	433,494,436	1,187,656
41	42	701,408,732	1,921,668
42	43	1,134,903,169	3,109,324
43	44	1,836,311,902	5,030,992
44	45	2,971,215,072	8,140,315
45	46	4,807,526,975	13,171,307
46	47	7,778,742,048	21,311,622
47	48	12,586,269,024	34,482,929
48	49	20,365,011,073	55,794,551
49	50	32,951,280,098	90,277,480
50	51	53,316,291,172	146,072,031
51	52	86,267,571,271	236,349,510
52	53	139,583,862,444	382,421,541
53	54	225,851,433,716	618,771,051
54	55	365,435,296,161	1,001,192,592
55	56	591,286,729,878	1,619,963,644
56	57	956,722,026,040	2,621,156,236
57	58	1,548,008,755,919	4,241,119,879
58	59	2,504,730,781,960	6,862,276,115
59	60	4,052,739,537,880	11,103,395,994
60	61	6,557,470,319,841	17,965,672,109
61	62	10,610,209,857,722	29,069,068,103
62	63	17,167,680,177,564	47,034,740,213
63	64	27,777,890,035,287	76,103,808,316
64	65	44,945,570,212,852	123,138,548,528

Pascal Row (0-indexed)	Pascal Row (1-indexed)	Cumulative Fibonacci ΣFn	Years (Cumulative)
65	66	72,723,460,248,140	199,242,356,844
66	67	117,669,030,460,993	322,380,905,373

Note. Cumulative Fibonacci values represent the running total obtained by summing all Fibonacci numbers up to the given row.